Graph-Based Path Selection and Power Allocation for DF Relay-Aided Transmission

Lu Lu, Dawei He, Qiqin Xie, Geoffrey Ye Li, and Xingxing Yu

***Abstract*—In this letter, we study path selection and power allocation for decode-and-forward relay-aided systems with multiple source, relay, and destination nodes. To take fairness among different links into account, we aim at maximizing the minimum source-relay-destination link rate under two scenarios depending on whether source nodes have their targeted destination nodes or not. For each scenario, graph-based algorithms are proposed to find the optimal solutions for cases with/without sum power constraints on relay nodes.**

***Index Terms*—Relay-aided transmission, graph theory, path selection, power control.**

I. INTRODUCTION

COOPERATIVE transmission and relay strategies have attracted extensive attention due to their benefits on improving coverage, spectral, and energy efficiency of wireless networks [2], [3]. Due to practical constraints, a half-duplex mode is normally used at the relay node where the relay cannot transmit and receive simultaneously [4]. In a practical system with several relay nodes, relay selection and power allocation are the most important issues and many existing works have studied systems with single or multiple source-destination pairs from different perspectives [5]–[13]. In general, relay selection and power allocation with multiple source, relay, and destination nodes are NP-hard and challenging [7]–[13]. To achieve better system performance, further study is necessary.

In this letter, we mainly focus on using graph theory tools to solve path selection and power allocation problems to maximize the minimum rate among source-to-relay or relay-to-destination for a system with multiple source, relay, and destination nodes. Two scenarios are considered: (1) Each source node can be paired with any destination node; (2) Each source node has its targeted destination node. For both scenarios, optimal algorithms are proposed to find the relay selection and power allocation solutions with or without the sum power constraint on relay nodes. The novelty of this letter is our objective functions and used graph based tools, which are different from other existing works.

II. SYSTEM MODEL

We consider a system with M source nodes, N relay nodes, and T destination nodes, as shown in Fig. 1. We focus on the scenario that the source nodes are far away from destination nodes and they cannot communicate directly without relay nodes. A half-duplex *decode-and-forward* (DF) relay is considered in [4]. The channels between the i-th source and the j-th relay and between the j-th relay to the k-th destination are denoted as $h_{i,j}^{(SR)}$ and $h_{j,k}^{(RD)}$, respectively. All channels are modeled as block Rayleigh fading channels.

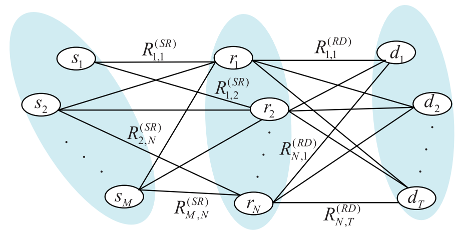

Fig. 1. A relay aided system with M sources, N relays, and T destinations.

To simplify the interference environment and to reduce implementation complexity, we assume each relay can only assist one source-to-destination link. Moreover, we assume relay nodes have been initially assigned orthogonal resources or spectrum bands. Our design will focus on assigning relays and their spectra for different source-to-destination links. A centralized system is assumed, where the control center knows *channel state information* (CSI) and allocates resources based on it. The resulting resource allocation performance can be treated as an upper bound for cases with imperfect CSI.

The rates from the i-th source to the j-th relay and from the j-th relay to the k-th destination, are $R_{i,j}^{(SR)}(P_i^{(S)}) = \frac{1}{2}\log(1 + \frac{P_i^{(S)}|h_{i,j}^{(SR)}|^2}{\sigma^2})$, and $R_{j,k}^{(RD)}(P_j^{(R)}) = \frac{1}{2}\log(1 + \frac{P_j^{(R)}|h_{j,k}^{(RD)}|^2}{\sigma^2})$, respectively, where $P_i^{(S)}$ and $P_j^{(R)}$ are the transmit power of the i-th source and the j-th relay, respectively. σ^2 is the noise power, and $\sigma^2 = 1$. To simplify analysis, we consider the case that the power constraints of the sources are independent and they will transmit at their maximum power, i.e., $P_i^{(S)} = P^{(S)}$. Then, $R_{i,j}^{(SR)}(P_i^{(S)})$ can be simplified as $R_{i,j}^{(SR)}$. For the relay nodes, we consider two different cases of power constraints. The first case is that the relay nodes are different equipment and the power constraint is the same as the source node, where $P_j^{(R)} = P^{(R)}$. The second case is that the relay nodes are different resources but belong to the same equipment, where the sum power cannot exceed the capability of the equipment, that is $\sum_j P_j^{(R)} \le P$, where P is the sum power threshold.

Based on the source-relay and relay-destination link rate, according to the cut-set bound theory [14], the achievable rate from the i-th source to the k-th destination through the j-th relay is

$$R_{i,j,k}^{(SRD)}(P_j^{(R)}) = \min\left\{R_{i,j}^{(SR)}, R_{j,k}^{(RD)}(P_j^{(R)})\right\}. \quad (1)$$

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L. Lu and G. Y. Li are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0250 USA (e-mail: lulu0528@gatech.edu).

D. He, Q. Xie, and X. Yu are with the School of Mathematiics, Georgia Institute of Technology, Atlanta, GA 30332-0250 USA.

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III. Problem Formulation

In this letter, we investigate how to assign relays to assist different source-to-destination links to maximize the throughput of the worst link [15]. Two system scenarios will be considered depending on whether sources have their targeted destinations or not.

We define a weighted graph by assigning rate on all the source-to-relay and relay-to-destination edges. As shown in Fig. 1, we assign $R_{i,j}^{(SR)}$ and $R_{j,k}^{(RD)}(P_j^{(R)})$ on the edge between the i-th source and the j-th relay and the edge between the j-th relay and the k-th destination, respectively. Furthermore, to simplify analysis, we assume that the numbers of the source, the relay, and the destination nodes are equal, i.e., $M = N = T$. The proposed algorithms can be extended to general cases by adding virtual nodes [16] or introducing admission control strategies [17]. When virtual nodes are added, zero weight will be put on edges linked to virtual nodes. When apply our proposed iterative algorithm in Section IV, consider non-zero minimum weight edge only where zero weight edges will be removed automatically.

A. Flexible Source-Destination Pairs

In this part, we consider the scenario that the destination for each source is flexible, which is suitable for the cases that source has information for all destinations or destination nodes can share information locally. We will assign source-to-destination pairs as well as a relay for each pair. Our problem can be treated as finding vertex/node disjoint paths based on graph theory [18]. We will use the throughput as the weight function and aim at finding disjoint paths solutions to maximize the minimum path weight. Using weights of source-to-relay and relay-to-destination edges, the path weight can be obtained based on (1).

Denote $\mathcal{D}$ as a feasible solution, which is a set of vertex disjoint paths and has N elements. $\mathcal{D}$ consists of two matchings, including one between source and relay nodes, denoted as $\mathcal{M}_1(\mathcal{D})$, and the other between relay and destination nodes, denoted as $\mathcal{M}_2(\mathcal{D})$, where

$$\mathcal{M}_1(\mathcal{D})=\left\{(s_i, r_j)|\exists d_k \text{ such that } (s_i, r_j, d_k) \in \mathcal{D}, j=1, \ldots, N\right\},$$

and

$$\mathcal{M}_2(\mathcal{D})=\left\{(r_j, d_k)|\exists s_i \text{ such that } (s_i, r_j, d_k) \in \mathcal{D}, j=1, \ldots, N\right\}.$$

Denote $M_{1,j} = (s_i, r_j) \in \mathcal{M}_1(\mathcal{D})$ and $M_{2,j} = (r_j, d_k) \in \mathcal{M}_2(\mathcal{D})$, as the pair linked to the j-th relay in $\mathcal{M}_1(\mathcal{D})$ and $\mathcal{M}_2(\mathcal{D})$, respectively. Then, the achievable rates of $M_{1,j}$ and $M_{2,j}$ are $R(M_{1,j}) = R_{i,j}^{(SR)}$ for $(s_i, r_j) \in \mathcal{M}_1(\mathcal{D})$ and $R(M_{2,j}, P_j^{(R)}) = R_{j,k}^{(RD)}(P_j^{(R)})$ for $(r_j, d_k) \in \mathcal{M}_2(\mathcal{D})$, respectively. The rate of the $s_i - r_j - d_k$ path in (1) can be expressed as

$$\min\{R(M_{1,j}), R(M_{2,j}, P_j^{(R)})\}. \tag{2}$$

Then, the resource allocation problem to maximize the minimum path weight can be expressed as

$$\max_{\mathcal{D}} \min_{\substack{M_{1,j}\in\mathcal{M}_1(\mathcal{D})\\ M_{2,j}\in\mathcal{M}_2(\mathcal{D})}} \min\{R(M_{1,j}), R(M_{2,j}, P_j^{(R)})\} \tag{3}$$

with an equal power assumption $P_j^{(R)} = P^{(R)}$ at all relay nodes, and

$$\max_{\mathcal{D}} \max_{\sum_j P_j^{(R)} \leq P} \min_{\substack{M_{1,j}\in\mathcal{M}_1(\mathcal{D})\\ M_{2,j}\in\mathcal{M}_2(\mathcal{D})}} \min\{R(M_{1,j}), R(M_{2,j}, P_j^{(R)})\} \tag{4}$$

with a sum power constraint at relay nodes, respectively.

Next, we will show that the problems in (3) and (4) can be decoupled into two one-hop problems. We take (4) as an example and (3) can be dealt with similarly.

Given $\mathcal{D}$, since $R(M_{1,j})$ is independent with $M_{2,j}$, $R(M_{2,j}, P_j^{(R)})$ is independent with $M_{1,j}$ and $\sum_j P_j^{(R)} \leq P$ only has impact on $R(M_{2,j}, P_j^{(R)})$, the minimal path weight is

$$\max_{\sum_j P_j^{(R)} \leq P} \min_{\substack{M_{1,j}\in\mathcal{M}_1(\mathcal{D})\\ M_{2,j}\in\mathcal{M}_2(\mathcal{D})}} \min\{R(M_{1,j}), R(M_{2,j}, P_j^{(R)})\}$$
$$= \min\left\{ \min_{M_{1,j}\in\mathcal{M}_1(\mathcal{D})} R(M_{1,j}),\ \max_{\sum_j P_j^{(R)} \leq P} \min_{M_{2,j}\in\mathcal{M}_2(\mathcal{D})} R(M_{2,j},P_j^{(R)}) \right\} \tag{5}$$

Then, the resource allocation problem (4) can be written as

$$\max_{\mathcal{D}}\min\left\{ \min_{M_{1,j}\in\mathcal{M}_1(\mathcal{D})} R(M_{1,j}),\ \max_{\sum_j P_j^{(R)} \leq P} \min_{M_{2,j}\in\mathcal{M}_2} R(M_{2,j},P_j^{(R)}) \right\} \tag{6}$$

As any two matchings, $\mathcal{M}_1$ and $\mathcal{M}_2$, uniquely define a feasible solution $\mathcal{D}$, (6) is equivalent to

$$\max_{\mathcal{M}_1,\mathcal{M}_2}\min\left\{ \min_{M_{1,j}\in\mathcal{M}_1} R(M_{1,j}),\ \max_{\sum_j P_j^{(R)} \leq P} \min_{M_{2,j}\in\mathcal{M}_2} R(M_{2,j},P_j^{(R)}) \right\} \tag{7}$$

Since $\mathcal{M}_1$ and $\mathcal{M}_2$ are independent with each other, to maximize the minimum of two terms in (7) is equivalent to maximize both of them independently. We can remove the subscript 1 and 2, and the solution for (7) can be found by solving

$$\max_{\mathcal{M}} \min_{M_j\in\mathcal{M}} \{R(M_j)\}, \tag{8}$$

and

$$\max_{\mathcal{M}} \max_{\sum_j P_j^{(R)} \leq P} \min_{M_j\in\mathcal{M}} \{R(M_j, P_j^{(R)})\}. \tag{9}$$

respectively. Note that, for the problem in (3), similar operations can be performed to decouple it to two subproblems in the form of (8).

B. Given Source-Destination Pairs

When the source-to-destination mapping is given, the original two-hop system can be modified as a one-hop system and also a bipartite graph, where one partition set consists of the source-destination pairs and the other consists of the relay nodes. We assume the targeted destination of the i-th source is the i-th destination. Denote $\mathcal{M}$ as any feasible matching. Then, our problem is a matching problem and can be expressed as (8) and (9) by changing the objective function as $R(M_{i,j}) = \min\{R_{i,j}^{(SR)}, R_{j,i}^{(RD)}(P^{(R)})\}$ for the case with and without sum power constraint.

IV. Path Selection and Power Allocation

In this section, we propose algorithms to solve the formulated path selection and power allocation problems formulated in the last section.

A. Flexible Source-Destination Pairs

For this scenario, we propose algorithms to solve the decoupled sub-problems in (8) and (9).

1) With Sum Power Constraints (9): Given a matching $\mathcal{M}$, the inner problem of (9),

$\max_{\sum_j P_j^R \le P} \min_{M_j \in \mathcal{M}} R(M_j, P_j^{(R)})$, has the solution that

$$P_{\mathcal{M},j}^* = \frac{\gamma_{\mathcal{M}}}{|h_{M_j}|^2}, \tag{10}$$

where $\gamma_{\mathcal{M}} = \frac{P}{\sum_{M_j \in \mathcal{M}} \frac{1}{|h_{M_j}|^2}}$, and $h_{M_j} = h_{j,k}^{(RD)}$. The solution is based on the fact that, for a given matching $\mathcal{M}$, the optimal power solution satisfies $|h_{M_1}|^2 P_{\mathcal{M},1}^* = |h_{M_2}|^2 P_{\mathcal{M},2}^* = \ldots = |h_{M_N}|^2 P_{\mathcal{M},N}^*$.

Then, the outer matching problem of (9) is equivalent to finding a resource allocation matching $\mathcal{M}$ for

$$\max_{\mathcal{M}} \gamma_{\mathcal{M}} = \frac{P}{\sum_{M_j \in \mathcal{M}} \frac{1}{|h_{M_j}|^2}}. \tag{11}$$

This problem can be solved by utilizing the Hungarian algorithm [18], [19]. First, we define a bipartite graph with N nodes in each partition set $\mathcal{V}_i$, $i = 1, 2$, and assign $-\frac{1}{|h_{j,i}^{(RD)}|^2}$ as the weight between nodes i and j from partition $\mathcal{V}_1$ to $\mathcal{V}_2$, respectively. Then, we utilize the Hungarian algorithm to find a maximal weight matching in the defined bipartite graph. The resulting matching is the solution for (9). The complexity of the Hungarian algorithm is $O(N^3)$. After the optimal matching is obtained, the optimal power for the j-th relay can be calculated according to (10).

2) Without Sum Power Constraint (8): To solve the max-min matching problem in (8), we can use the idea from [20]. The procedure is as follows. First, find a perfect matching of the bipartite graph. Determine the minimum weight edge of the perfect matching. Delete all edges that have less or equal weight of the minimum weight edge. Try to find augmenting path with respect to the new matching. If no augmenting path can be found, the perfect matching from the previous step is our solution. Otherwise, conduct the augmentation procedure and find a new perfect matching. Repeat the procedure on the new matching until no augmenting path can be found. More detail can be found in [1]. The algorithm leads to global optimal solution and has complexity $O(N^{5/2} \log N)$.

B. Fixed Source-Destination Pairs

When there is no sum power constraint, the weight for each edge can be calculated independently and thus, the algorithm from Section IV-A2 can be used directly.

Next, we mainly focus on the case with sum power constraint. In this case, the weight function $R(M_{i,j}) = \min\{R_{i,j}^{(SR)}, R_{j,i}^{(RD)}(P_j^{(R)})\}$ relates to the power allocation result, which depends on the matching and makes the problem more complicated. Hence, we define two bipartite graphs $\mathcal{G}_{s,r}(\mathcal{V}_S, \mathcal{V}_R, \mathcal{E}_{s,r})$ and $\mathcal{G}_{r,d}(\mathcal{V}_R, \mathcal{V}_D, \mathcal{E}_{r,d})$, where $\mathcal{V}_S$, $\mathcal{V}_R$, and $\mathcal{V}_D$ are the sets of source, relay, and destination nodes, respectively, and $\mathcal{E}_{s,r}$ and $\mathcal{E}_{r,d}$ are the sets of source-to-relay and the relay-to-destination edges, respectively. Weights are defined as in Section III-A.

In this case, for a given relay-to-destination matching, the source-to-relay matching is determined. Given a matching $\mathcal{M}$, one optimal power solution can be found as in Section IV-A1, denoted as $P_{\mathcal{M},j}^*$. Note that, the solution from Section IV-A1 is only one of many optimal power solutions and its sum power consumption is the maximum among all optimal solutions. We will adjust the power at the final step. Based on the power allocation results and the matching $\mathcal{M}$, the minimum weights among the relay-destination links and the source-relay links are defined as

$$R_{\mathcal{M},r,d}^* = \frac{1}{2}\log_2\left(1 + \frac{P_{\mathcal{M},j}^* |h_{j,k}^{(RD)}|^2}{\sigma^2}\right) \text{ for } M_{k,j} \in \mathcal{M}. \tag{12}$$

and

$$R_{\mathcal{M},s,r}^* = \min_{M_{i,j} \in \mathcal{M}} \left\{\frac{1}{2}\log_2\left(1 + \frac{P^{(S)} |h_{i,j}^{(SR)}|^2}{\sigma^2}\right)\right\}. \tag{13}$$

Accordingly, the minimum link weight based on $\mathcal{M}$ is

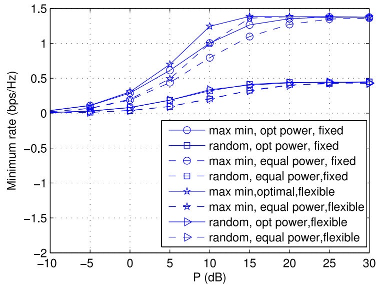
$$R_{\mathcal{M}}^* = \min\left\{R_{\mathcal{M},s,r}^*, R_{\mathcal{M},r,d}^*\right\}. \tag{14}$$

Next, we introduce our iterative algorithm to find the global optimal matching solution. We start with an optimal relay-to-destination matching. Then, the source-to-relay matching is determined. If the minimum edge is on the relay-to-destination side, then the algorithm stops. If the minimum edge is on the source-to-relay side, we remove edges and check if we can still find a perfect matching. If no, current solution is optimal. Otherwise, we repeat the process. More detailed procedure is as follows, where $\mathcal{M}^*$ denotes the optimal matching solution.

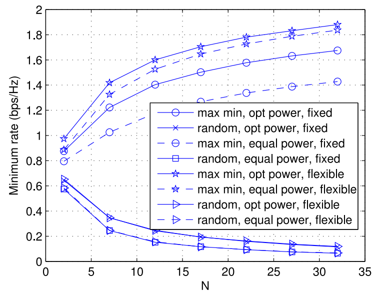
1) Initialization: We start with an extreme case by considering the relay-to-destination matching only. This is step 0. Find the relay-to-destination matching as in Section IV-A1, denoted as $\mathcal{M}_0$. If the minimum weight among the relay-to-destination links, $R_{\mathcal{M}_0,r,d}^*$, is smaller than the minimum weight of the corresponding source-to-relay links, $R_{\mathcal{M}_0,s,r}^*$, $\mathcal{M}^* = \mathcal{M}_0$ and the algorithm stops. Otherwise, go to the next step.

2) Iteration: For step k, denote the source and the relay nodes corresponding to the minimum weight edge from the previous step $\mathcal{M}_{k-1}$ as $s_{\hat{i}_{k-1}}$ and $r_{\hat{j}_{k-1}}$, respectively. To find a better solution, we can delete all edges that have equal or smaller weights than $M_{\hat{i}_0,\hat{j}_0}$ from the graph. The corresponding relay-to-destination links are deleted as well. The remaining source-to-relay and relay-to-destination edge sets are denoted as $\mathcal{E}_{s,r,k}$ and $\mathcal{E}_{r,d,k}$, respectively. Find an optimal perfect matching of $\mathcal{G}_{r,d,k}(\mathcal{V}_R, \mathcal{V}_D, \mathcal{E}_{r,d,k})$ as in Section IV-A1, denoted as $\mathcal{M}_k$. If $\mathcal{M}_k$ is not a perfect matching, $\mathcal{M}^* = \mathcal{M}_{k-1}$. Otherwise, if $R_{\mathcal{M}_k,r,d}^* \le R_{\mathcal{M}_{k-1},s,r}^*$ the solution cannot be improved and set $\mathcal{M}^* = \mathcal{M}_{k-1}$. If $R_{\mathcal{M}_k,r,d}^* > R_{\mathcal{M}_{k-1},s,r}^*$, check the minimum weight of the source-to-relay links based on $\mathcal{M}_k$, $R_{\mathcal{M}_k,s,r}^*$. Similar to the previous step, if $R_{\mathcal{M}_k,r,d}^* \le R_{\mathcal{M}_k,s,r}^*$, $\mathcal{M}_k$ is the optimal solution, that is $\mathcal{M}^* = \mathcal{M}_k$. Otherwise, conduct the edge deleting procedure and repeat the procedure until the optimal solution is found. The complexity of the proposed algorithm is $O(N^5)$.

As mentioned before, optimal power allocations based on $\mathcal{M}^*$ are not unique; so we adjust the power allocation based on the optimal matching $\mathcal{M}^*$ here. Since the minimum link value is also bounded by the source-to-relay links, we can reduce the minimum relay-to-destination weight to the minimum source-to-relay weight by decreasing the power consumption level



(a) The impact of sum power.



(b) The impact of node number.

Fig. 2. Two-hop worst link performance.

without impacting the final solution. Thus, the optimal solution that has the minimum sum power consumption can be expressed as

$$P^*_{\mathcal{M}^*,j} = \frac{\gamma_{\mathcal{M}^*}}{|h^{(RD)}_{j,k}|^2} = \frac{P^{(S)} \min_{M_{k,j}\in\mathcal{M}^*} |h^{(SR)}_{k,j}|^2}{|h^{(RD)}_{j,k}|^2}.$$

V. Numerical Results

In this section, we present numerical results to show the performance of the proposed algorithms. For comparison, results using random matching are also provided, where the matching is chosen randomly and the optimal power allocation is based on algorithms in Section IV.

Figs. 2(a) and 2(b) show the impact of the sum power and the node number on the minimum rate performance, respectively. The average node power is set to be $P^{(R)} = P^{(S)} = 5dB$ and the sum power is $P = NP^{(R)}$.

Fig. 2(a) shows the results with $N = 6$. With the increase of the sum power consumption of both relay nodes, P, the minimum rate increases first and then converges. When the sum power at the relay is small, the minimum link rate is dominated by the relay-to-destination link. The minimum rate of the relay-to-destination link increases with the sum power, leading to the worst source-relay-destination link performance improvement. When the sum power at relay nodes is large enough, the worst link performance is dominated by the source-to-relay link and the worst link performance converges.

Fig. 2(b) shows the impact of the node number. Based on our proposed algorithms, the minimum source-relay-destination link performance increases with the node number while it decreases based on the random matching. The performance gain introduced by our proposed algorithms to the random algorithms increases with the node number. Comparing the scenarios with flexible and fixed source-destination pairs, the former scenario, which has higher level of flexibility, can provide better minimum link performance.

VI. Conclusion

We propose graph theory based algorithms to find optimal path selection and power allocation solutions to maximize the minimum path rate. Algorithm complexities are given. The more the nodes in the system, the higher performance gain we can provide compared to the random matching. Moreover, the system with flexible source-destination pairs can provide better minimum path rate performance compared to the fixed source-destination scenario.

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